1. Introduction

Special functions are particular mathematical functions which have more or less established names and notations due to their importance in mathematical analysis, functional analysis, physics, or other applications. Many special functions appear as solutions of differential equations, integrals of elementary functions and more recently Fractional differential equations.

Special Functions and Fractional Calculus is the most recently interestedarea for both mathematicians and applied scientists. This is due to their role as solutions of fractional order differential and integral equations, as the better mathematical models of phenomena of various physical, engineering, automatization, chemical, biological, Earth science, economics etc. nature. Our approach is based on the use of Generalized Fractional Calculus (GFC) operators. Some prominent examples include the gamma function, generalized hypergeometric function, Whittaker function, and Meijer G-function and many more. Generalized hypergeometric functions include the (Gaussian) hypergeometric function and the (confluent) hypergeometric function as special cases, which in turn have many particular special functions as special cases, such as elementary functions, Bessel functions and the classical orthogonal polynomials.

Some basic definition of special functions and fractional derivative, fractional integral operators are defined here:

1.1 Generalized hypergeometric function

The natural generalization of the Gauss's hypergeometric function ${}_2F_1$ is called the generalized hypergeometric series ${}_pF_q$(where $p, q \in N_0$)and are defined by

$$ {}_p\boldsymbol{F}_q\left[\begin{matrix}\boldsymbol{\alpha_1, \alpha_2, \dots., \alpha_p; z}\\ \boldsymbol{\beta_1, \beta_2, \dots. \beta_q}\end{matrix}\right] = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n (\alpha_2)_n \dots (\alpha_p)_n}{(\beta_1)_n (\beta_2)_n \dots (\beta_q)_n} \frac{z^n}{n!}, \qquad \textbf{...(1)} $$

In which no denominator parameter β_j can be zero or a negative integer. If any numerator parameter α_i in (1) is zero or negative integers, the series terminates itself. An application of elementary ratio test to the power series on the right in equation (1) shows the same:

a) If $p \leq q$ the series converge absolutely for every finite z;
b) If $p = q + 1$the series converge absolutely when $|z| < 1$ and divergence for $|z| > 1$;
c) If $p > q + 1$, the series diverge for $z \neq 0$. If the series terminates, there is no question of convergence and the conclusions (b) and (c) will not apply.
d) If $p = q + 1$, the series in (1) is absolutely convergent on the circle $|z| = 1$, if $Re(\sum_{j=1}^{q} b_j - \sum_{i=1}^{p} a_i) > 0$,

Here $\mathbb{C}$, R, $\mathbb{Z}$ and N are the sets of complex numbers, real numbers, positive real numbers, integers and positive integers respectively.

and $(\boldsymbol{\lambda})_{\boldsymbol{n}}$is Pochhammer symbol ($for\ \lambda \in \mathbb{C}$) defined as:

$$ (\boldsymbol{\lambda})_{\boldsymbol{n}} = \frac{\Gamma(\boldsymbol{\lambda+n})}{\Gamma(\boldsymbol{\lambda})} \ (\boldsymbol{\lambda} \in \mathbb{C}/\mathbb{Z}_{\boldsymbol{0}}^{-}), $$

$$= \begin{cases} 1 & (n = 0) \\ \lambda(\lambda + 1) \dots \dots (\lambda + n - 1) & (n \in N) \end{cases}$$

1.2 Beta function

$$B(\alpha, \beta) = \int_0^1 (t)^{\alpha-1}(1-t)^{\beta-1} dt \quad (\Re(\alpha) > 0, \Re(\beta) > 0)$$

Or

$$B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$$

1.3 Incomplete gamma functions

In mathematics, the upper and lower incomplete gamma functions are types of special functions which arise as solution to various mathematical problems.

Their respective names stem from their integral definitions, these functions are the gamma function but with different variable integral limits. The gamma function is defined as an integral from zero to infinity. While the lower and upper incomplete gamma functions are defined as an integral from zero to a variable upper limit and variable lower limit to infinity as the upper limit respectively

The upper incomplete gamma function defined as:

$$\Gamma(s, x) = \int_x^\infty t^{(s-1)} e^{-t} \text{ dt}$$

while the lower incomplete gamma function is defined as:

$$\gamma(s, x) = \int_0^x t^{(s-1)} e^{-t} \text{ dt}$$

The lower and upper incomplete gamma functions, as defined above for real positive *s* and *x*. Same as developed into holomorphic functions, with respect to both *x* and *s*, defined for almost all combinations of complex numbers *x* and *s*. With the help of these functions properties of the real incomplete gamma functions extend to their holomorphic counterparts.

1.4 Incomplete Pochhammer symbol

Using the concept of familiar incomplete gamma functions Srivastava et al. [26] introduced the incomplete Pochhammer symbol that leads to a natural generalization and decomposition of a class of hypergeometric and other related functions; same are potentially useful in closed-form representation of definite and semi-infinite integrals. Application of these functions can be found in the field of communication theory, probability theory and groundwater pumping modelling etc.

1.5 Incomplete generalized hypergeometric functions

Recently some fundamental properties and characteristics of a family of potentially useful incomplete hypergeometric functions are studied. Using this development many author investigated several further properties of these incomplete hypergeometric functions and some general classes of incomplete hypergeometric polynomials associated with them.

1.6 Incomplete Srivastava's triple hypergeometric functions

In 2012, Shrivastava et al. [26, 27], introduced the incomplete Pochhammer symbols by using familiar incomplete gamma functions $\gamma(s, x)$ and $\Gamma(s, x)$, and defined incomplete generalized hypergeometric function in terms of them such as

$$ {}_p\gamma_q\left[\begin{matrix}(A_p,x);\\ B_q;\end{matrix}z\right]+\ {}_p\Gamma_q\left[\begin{matrix}(A_p,x);\\ B_q;\end{matrix}z\right]=\ {}_pF_q\left[\begin{matrix}A_P;\\ B_q;\end{matrix}z\right],\ \ (p,q\in N_0) $$

In 2013 Cetinkaya [4] introduced the incomplete second Appell hypergeometric functions γ_2 and Γ_2 such as

$$ \gamma_2[(\alpha,x),\beta_1,\beta_2;\gamma_1,\gamma_2;x_1,x_2]=\sum_{m,p=0}^{\infty}\frac{(\alpha;x)_{m+p}(\beta_1)_m(\beta_2)_p}{(\gamma_1)_m(\gamma_2)_p}\frac{x_1^m}{m!}\frac{x_2^p}{p!}, $$

and

$$ \Gamma_2[(\alpha,x),\beta_1,\beta_2;\gamma_1,\gamma_2;x_1,x_2]=\sum_{m,p=0}^{\infty}\frac{[\alpha;x]_{m+p}(\beta_1)_m(\beta_2)_p}{(\gamma_1)_m(\gamma_2)_p}\frac{x_1^m}{m!}\frac{x_2^p}{p!}, $$

Motivated by the above-mentioned works Choi et al. [7] presented two incomplete Srivastava's triple hypergeometric functions by using the incomplete Pochhammer symbols such as

For $\alpha,\beta_1,\ldots\ldots,\beta_n\in C$ and $\gamma_1,\ldots\ldots,\gamma_n\in C\backslash Z_0^-$,

$$ \gamma_B^H[(\alpha,x),\beta_1,\beta_2;\gamma_1,\gamma_2,\gamma_3;x_1,x_2,x_3]+\Gamma_B^H[(\alpha,x),\beta_1,\beta_2;\gamma_1,\gamma_2,\gamma_3;x_1,x_2,x_3] $$
$$ =H_B[\alpha,\beta_1,\beta_2;\gamma_1,\gamma_2,\gamma_3;x_1,x_2,x_3] $$

and certain properties were also investigated by them, like integral representations, derivative formula, and reduction formula and recurrence relation.

1.7 Riemann-Liouville fractional integral operator

The Riemann-Liouville fractional integral operator are defined by samkoet. al. [[25], Sec.5.1].

$$ I_{0+}^{\alpha}\ f\ (x)\ =\ \frac{1}{\Gamma(\alpha)}\int_0^{\infty}(x-s)^{\alpha-1}\ f\ (s)\ ds,\ \mathrm{Re}(\alpha)>0. $$

1.8 Weyl type fractional integral operator

The Weyl type fractional integral operator are defined by samko et. al. [[25], Sec.5.1].

$$ I_{0-}^{\alpha}\ f\ (x)\ =\ \frac{1}{\Gamma(\alpha)}\int_x^{\infty}(x-s)^{\alpha-1}\ f\ (s)\ ds,\ \mathrm{Re}(\alpha)>0. $$

1.9 Fractional Derivative operator

(i) The fractional derivative of order $\alpha\in(n-1,n)$ of a function $f(x)$ given by Riemann-Liouville [16].

$$ \frac{d^{\alpha}}{dx^{\alpha}}f\ (x)=D^n\ I^{n-\alpha}\ f\ (x)\ ,\ \ D\ \equiv\frac{d}{dx} $$

(ii) The fractional derivative of order $\alpha \in (n-1, n)$ of a function $f(x)$given by Caputo [3].

$$D^{\alpha} f(x) = I^{n-\alpha} D^{n} f(x), \quad D \equiv \frac{d}{dx}$$

2. Review of literature

In 1994, Choudhary et al.[10] introduced new functions as generalization of the incomplete gamma functions. The functions are useful in heat conduction, probability theory and in the study of Fourier and Laplace transforms. They investigated the asymptotic behaviour, Laplace transforms, decomposition formula, integral representations, convolutions, recurrence relations and differentiation formula of these functions. When these functions were applied in evaluation, Inverse Laplace transforms to the definite integrals and to the infinite series of exponential functions was also given.

In 1997, Choudhary et al.[12] introduced extensions $\gamma_v(\alpha, x; b)$ and $\Gamma_v(\alpha, x; b)$ of the generalized Gamma functions $\gamma(\alpha, x; b)$ and $\Gamma(\alpha, x; b)$. These extensions were found useful in the representations of the Laplace and *K*-transforms ([17], page 279) of a class of functions. They also defined a generalization of the inverse Gaussian distribution. The cumulative and the reliability functions of the generalized inverse Gaussian distribution were expressed in terms of these functions. Some useful properties of the functions were also discussed.

In 1997, Choudhary et al.[13] proved the generalized incomplete gamma functions andtheir extensions which were mutually related through integral and differential representations.

In 2004, Choudhary et al.[15] introduced an extension of the beta function by introducing an extra parameter, and same was applied to extend the hypergeometric and confluent hypergeometric functions. They established integral representation, Mellin transform, differentiation formula, Mellin Barne's contour integral representation for these extended hypergeometric function, other than this they also presented Summation formula and asymptotic behavior, transformation formula, recurrence relation.

In 2007, Khan et al.[17] investigated the evaluations of certain Euler type integrals. Further, they established a theorem on extended beta function and same was applied to obtain evaluations of some known integrals. They derived a number of new results as applications of these evaluated Euler type integrals.

In 2011, Ozergin et al.[22] presented generalizations of gamma, beta and hypergeometric functions. Some recurrence relations, transformation formulas, operation formulas and integral representations were obtained for these new generalized functions.

In 2013, Ahmad et al.[2] defined the complete triple hypergeometric functions of the second order which were denoted by each of these functions. They also investigated extensively in many different ways, finding the problem of their integral representations of one kind or other.

In 2013, Choi et al.[5] gave the reduction formulas. Which have naturally been investigated with the help of certain general single or multiple hypergeometric functions.

They established some very interesting reduction formula's for the Srivastava's triple hypergeometric function.

In 2014, Choi et al.[6] introduced the incomplete Pochhammer symbol and studied some fundamental properties and characteristics of a family of potentially useful incomplete hypergeometric functions. They introduced the incomplete Lauricella function $\gamma_D^{(n)}$ and $\Gamma_D^{(n)}$ with n variables. After that they investigated certain properties of the incomplete Lauricella functions, for example, their various integral representations, differential formulas and recurrence relations.

In 2014, Srivastava [28], studied many interesting fundamental properties and characteristics of a certain pair of useful families of the so-called generalized incomplete hypergeometric functions. Then many closely-related works dealing with notable developments involving various classes of generalized hypergeometric function and generalized hypergeometric polynomials which were defined by incomplete families of Pochhammer symbol. In the sequel of earlier works, they derived two remarkably distinct classes of hypergeometric generating functions and then applied each of them in deducting a family of linear, bilinear and bilateral generating functions, they also considered various (known or new) special cases and consequences of the results.

In 2015, Agrawal et al.[1] introduced extensions of some classical special functions, for example, beta function and generalized hypergeometric functions have been actively investigated and found diverse applications. They proved some properties on generalize Appell's hypergeometric functions of two variables and Lauricella's hypergeometric function of three variables by using the extended generalized beta type function.

In 2015, Lin et al.[19] introduced an incomplete Pochhammer symbols and their applications to hypergeometric and related functions, integral transforms introduced and initiated the study of many interesting fundamental properties and characteristics of a certain pair of potentially useful families of the so-called generalized incomplete hypergeometric functions. Ever since then there have been appearance of many closely-related works dealing essentially with notable developments involving various classes of generalized hypergeometric functions and generalized hypergeometric polynomials, which are defined by means of the corresponding incomplete and other novel extensions of the familiar Pochhammer symbol. Here, in this sequel to some of their earlier work, they derived several general families of hypergeometric generating functions by applying Srivastava's Theorem.

In 2015, Yang et al.[30] introduced Local Fractional Integral Transforms and their applications provide information on how local fractional calculus has been successfully applied to describe the numerous wide spread real-world phenomena in the fields of physical sciences and engineering sciences. Same involved non-differentiable behaviours. These methods of integral transforms via local fractional calculus have been used to solve various local fractional ordinary and local fractional partial differential equations.

In 2018, Choi et al.[8] introduced the family of incomplete Srivastava's triple hypergeometric functions. In continuation, they investigated several properties like integral representation, transformation and reduction formula etc. for each of these incomplete functions.

In 2017, Mohammed et al.[21] presented generalization and extension of beta function with three and four parameters. This new beta function gives upto 4! derivative formulas

of new function including Euler's beta function and generalized beta function, their objective was to extend the domain of intersection of beta and gamma function.

In 2017, Pablo I. Pucheta [23] introduced a new and modified extended version of classical gamma, beta functions. Some basic properties were studied, such as the integral representations; functional relations and he also evaluated the action of the Mellin transform of the new modified extended beta function.
In 2017, Mubeen et al. [20], used various extended beta functions which were investigated earlier. They introduced a (p, q) –extended beta function by using the product of two confluent hypergeometric functions as a kernel and investigated its several properties.
In 2018, Rahman et al.[24] introduced a further extension of extended (p,q)-beta function by considering two Mittag-Leffler functions in the kernel. They investigated various properties of this newly defined beta function such as integral representations, summation formulas and Mellin transform and its inverse. They also explained extended beta distribution and its mean, variance and moment generating function with the help of extension of beta function. Establishing an extension of extended (p,q)-hypergeometric and (p,q)-confluent hypergeometric functions by using the extension of beta function. Various properties of newly defined extended hypergeometric and confluent hypergeometric functions involving extended (p,q)-beta function such as integral representations, Mellin transformations, differentiation formulas, and transformation and summation formulas are investigated.

In 2018, Shadab et al.[29] were established extended beta function involving Mittag-Leffler function with one parameter. They discussed the properties like integral representation, summation formulas and beta distribution for this extended beta function. They also derived extended hypergeometric function and confluent hypergeometric function and their properties. The graphical representation of the function for different value of Mittag-Leffler parameters was also discussed.

In 2018, Choi et al.[9] introduced two Gaussian hypergeometric summation formulas by specialising some known formulas. Applying these formulas they were able to derived two known summation formulas.

3. Objective

- To develop generalized hypergeometric functions by using incomplete gamma functions and beta functions.
- To investigate some statistical distributions involving extended gamma and beta functions.
- To explore the role of fractional calculus in modelling and simulation.
- To study some properties for extended hypergeometric functions.
- To develop generalized fractional derivative and fractional integral operators.

4. Plan of work and Methodology

- We may study incomplete gamma and beta functions in a form of generalized hypergeometric function.
- We propose to represent two Gaussian hypergeometric summation formulas by specialising some known formulas. Applying these we derive two known summation formulas.

- We may use incomplete Pochhammer ratios that are defined in terms of the incomplete beta function. With the help of these incomplete Pochhammer ratios we are able to introduce new incomplete Gauss, confluent hypergeometric and investigate several properties of them such as integral representations, derivative formulas, transformation formulas and recurrence relations.

- We propose explore some fractional differential equations in the field of modelling and simulation (Epidemic model, Compartment model, Voltera model and many more).
- We propose some statistical distributions involving extended gamma and beta function and represent there result with the help of mathematical software.
- We may introduce extended incomplete Riemann-Liouville fractional derivative operators in other field.

5. References

[1] Agrawal, P., J. Choi and S. Jain(2015), Extended hypergeometric functions of two and three variables, *Commun. Korean Math. Soc. 30*, **4**, 403-414.

[2] Ahmad, S. (2013), Hypergeometric functions of three variables in terms of integral representations, *IOSR Journalof Mathematics*, **8**, 67-73.

[3] Caputo, M. (1966). Linear models of dissipation whose Q is almost frequency independent. Annals of Geophysics, vol. 19, issue 4, 383-393.

[4] Çetinkaya, A.(2013), The incomplete second Appell hypergeometric functions, *Applied Mathematics and Computation,* **219**, 8332–8337.(title of the article)

[5] Choi, J., X. Wang and A.K. Rathi (2013), A reducibility of Srivastava's triple hypergeometric series f(3) [x, y, z], *Korean Math. Soc.,* **28**, 297–301.

[6] Choi, J., R.K. Parmar and P. Chopra (2014), The incomplete lauricella and first appell functions and associated properties, *Honam Mathematical*, **36**, 531–542.

[7] Choi, J., R.K. Parmar and P. Chopra (2016), The incomplete Srivastava's Triple Hypergeometric Functions γ_B^H and Γ_B^H, *Filomat,* **30** (issue 7), 1779–1787.

[8] Choi, J., R.K. Parmar and P. Chopra (2018), The incomplete Srivastava's Triple Hypergeometric Functions γ_B^H and Γ_B^H,*Miskolc Mathematical Notes,* **19**, 191-200.

[9] Choi, J., A.K Rathi(2018), Two Gaussian hypergeometric summation formula with an application, *Far East Journal of Mathematical Sciences,* **104**, 1-5.

[10] Choudhary, A.M. and S.M. Zubair (1994), Generalized incomplete gamma functions with applications, *Journal of Computational and Applied Mathematics, **55**,* 99-123.

[12] Choudhary, A.M. and S.M. Zubair (1997), On an extension of generalized incomplete Gamma functions with applications, *The Anziam Journal,* **37**, 392-475.

[13] Choudhary, A.M. and S.M. Zubair (1997), On a connection between the generalized incomplete gamma functions and their extensions**,** *The Anziam Journal,* **38**, 581-589.

[14] Choudhary, A.M., A. Qadir, H. M. Srivastava and R. B. Paris (2004), Extended Hypergeometric and Confluent Hypergeometric functions, *Appl. Math. Comput.*, **159**, 589-602.

[15] Choudhary, A.M, A. Quadir, H.M. Shrivastava and R.B Paris (2004), Notes on Generalized Hypergeometric and Confluent Hypergeometric Function, *International Journal of Mathematical Analysis and Applications,* **2**, 47-61.

[16] Gorenflo, R., Mainardi, F., (2008). Fractional calculus: integral and differential equations of fractional order. arXiv preprint arXiv:0805.3823.

[17] Khan, S., B. Agrawal, M. A. Pathan and F. Mohammad (2007), Evaluations of certain Euler type integrals, *Appl. Math. Comput*., **189**, 1993-2003.

[19] Lin, S., H. M. Srivastava and M. Wong (2015), Some Applications of Srivastava's Theorem involving a Certain Family of Generalized and Extended Hypergeometric Polynomials, *Filomat,* **29**, 1811-1819.

[20] Mubeen, S., G. Rahman, K. S. Nisar, J. Choi. and M. Arshad (2017), An extended beta function and its properties, *Far East Journal of Mathematical Sciences*, **102**, 1545-1557.

[21] Mohammed, A. and L. Bachioua (2017), On Extension of Euler's Beta Function, *Journal of Applied Mathematics & Bioinformatics*, **7**, 1-11.

[22] Ozergin, E., M.A. Ozarslan and A. Altin (2011), Extension of gamma, beta and hypergeometric functions, *Journal of Applied Computational and Mathematics,* **235,** 4601-4610.

[23] Pucheta P.I (2017), A new extended beta function, *International Journal of Mathematics And its Applications*, **5**, 255-260.

[24] Rahman, G., G. Kanwal, K. S. Nisar and A. Ghaffar (2018), A New Extension of Beta and Hypergeometric Functions, *Preprints*, 2018010074 (doi: 10.20944/preprints201801.0074.v1

[25] Samko, S.G., Kilbas, A.A., Marichev, O.I. (1993). Fractional Integrals and Derivatives: Theory and Applications, *Gordon & Breach*.

[26] Shrivastava, H.M, M.A Choudhary and R. Agrawal (2012), The incomplete pochhammer symbols and their applications to hypergeometric and related functions, *Integral Transforms and Special Functions,* **23,** 659–683.

[27] Srivastava, R., R. Agarwal and S. Jain (2012), A Family of the Incomplete Hypergeometric Functions and Associated Integral Transform and Fractional Derivative Formulas, Filomat, 31, 125-140

[28] Srivastava, S. (2014), Some Classes of Generating Relations Associated with a Family of the Generalized Gauss Type Hypergeometric Functions, *Applied Mathematics and Computation***, 243**, 132-137.

[29] Shadab, M., S. Jabee and J. Choi (2018), An extended beta function and its applications, *Far East Journal of Mathematical Sciences,* **103**, 235-251.

[30] Yang, X.J., D. Baleanu and H. M. Srivastava (2015), Local Fractional Integral Transforms and Their Applications., *Academic Press.*